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Mhd Flow of Visco-Elastic Fluid with Radiative Heat Transfer and Temperature Dependent Heat Source

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Abstract: *An analysis of visco-elastic mhd free convective flow and heat transfer with radiation and temperature dependent heat source confined between two vertical wavy walls is presented where one wall is isothermal and the other is adiabatic. The equations governing the flow field and heat transfer are solved by perturbation technique by assuming that the flow consists of a mean part and a perturbed part as the walls are purely sinusoidal. Expressions for velocity, temperature, skin friction coefficients at both the walls and pressure drop are obtained. Expressions for the zeroth-order and first order velocity, temperature, skin-friction, heat transfer coefficient at the walls and pressure drop are obtained. The first order velocity, skin friction coefficients at both the walls and pressure drop have been presented graphically to observe the visco-elastic effects in combination of other flow parameters involved in the solution.*
Key Words: *Grashof Number, mhd, perturbation technique, Prandtl Number, Visco-elastic,*

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I. Introduction

Analysis of fluid over a wavy wall is widely studied because of its application in different areas such as transpiration, cooling of re-entry vehicles and rocket boosters, cross hatching on ablative surfaces and film vaporization in combustion chambers.The radiation effects play an important role when the surrounding temperature of a fluid is high, and this situation occurs in space technology. In such cases the investigators have to consider the effects of radiation and free convection. Such studies were presented by Cess [1], Arpaci [2], Cheng and Ozisik [3], Hasegawa et al. [4], Hossain and Takhar, [7, 8], Hossain et al. [9], Tak and Kumar [10] and Mohamed et al. [11] in case of steady flows. In case of unsteady flows Raptis and Perdikis [12] have studied the flow past an accelerated plate by solving the governing equations numerically. Raptis et al. [6] have investigated hydro magnetic free convection flow through porous medium between two parallel plates. Ganeshan et al. [13] have analyzed the effects of radiation and free convection using Rosseland approximation defined in Brewster [14] for an impulsively started infinite vertical isothermal plate. A linear analysis of compressible boundary layer flows over a wavy wallhas been presented by Lekoudis et al. [15]. Shankar and Sinha [16] have studied the Reyleigh problem for a wavy wall in detail and found that the importance of the waviness of the wall ceases quickly as the liquid is dragged along the wall at low Reynolds numbers, while the effects of viscosity are confined to a thin layer in a neighbourhood of the wall at large Reynolds numbers. Bordner [17] has presented the non-linear analysis of laminar boundary layer flow over a periodic wavy surface applying suitable orthogonal transformations to transform the wavy surface to flat one. Bordner has found that some non-linear terms in the disturbance boundary layer equations are of first order if the wave amplitude and disturbance sub-layer thickness are comparable in magnitude. He has also found the non-linear effects to be confined to the thin sub-layer adjacent to the wavy surface. The effects of small amplitude wall waviness upon the stability of the laminar boundary layer have been studied by Lessen and Gangwani [18]. A modified slip boundary conditions to represent the effect of small roughness-like (slightly wavy) perturbations to an otherwise plane fixed wall which is acting as a boundary to steady laminar flow of a viscous fluid have been obtained by Tuck and Kouzoubov [19]. In all these cases the authors have taken the wavy walls to be horizontal. The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls has been studied by Patidar and Purohit [20]. Vajravelu and Sastri [21] and Das and Ahmed [22] have studied the problem of free convective flow of a viscous incompressible fluid with heat transfer confined between a vertical wavy wall and a flat wall. Sharma [23] has studied fluctuating thermal and mass diffusion on unsteady free convective flow past a vertical plate in slip- surface with heat source/sink and viscous dissipation.Tak and Kumar [24] have analysed the MHD free convection flow with viscous dissipation in a vertical wavy channel. Muthucumaraswamy and Chandrakala [25] have studied the radiation heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Nigam and Singh [26] have investigated the Heat transfer by Laminar flow between parallel plates under the action of transverse magnetic field. Mahdy et al. [27] have studied the problem of natural convection from a vertical wavy plate embedded in porous media for power law fluids in presence of magnetic field. Mahdy [28] has studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media. El-Aziz [29] has studied radiation effect on the flow and heat transfer over an unsteady stretching sheet. Suneethaet al. [30] have analysed radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink. Kumar [31] has studied the effect of heat transfer and radiation on a MHD free convective flow confined between two vertical wavy walls. Basu et al. [32] have investigated radiation and mass transfer effects on transient free convection flow of dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. Reddy and Reddy [33] have analysed mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium. Sandeep et al. [34] have studied the effect of radiation and chemical reaction on transient MHD free convective flow over a vertical plate through porous media. Bala et al. [35] have analysed the radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source. Choudhury and Das [36, 37, 38, 40, 41, 42, 43, 44, 45,] have analyzed some problems of physical interest in this field.39. Rao et al. [39] have investigated the chemical effects on an unsteady MHD free convection fluid past a semi-infinite vertical plate embedded in a porous medium with heat absorption.

The constitutive equation for Walters liquid (Model B′) is

$$\sigma^{ik} = -pg^{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik} \quad (1)$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g^{ik} is the metric tensor of a fixed co-ordinate system x^i, v^i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}{}_{,m} - v^i{}_{,m}\, e^{im} - v^i{}_{,m}\, e^{mk} \quad (2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^i{}_{,k} + v^k{}_{,i} \quad (3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \ \text{ and } \ k_0 = \int_0^\infty \tau N(\tau)d\tau \quad (4)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walter [46, 47]. This idealized model is a valid approximation of Walters liquid (Model B′) taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)d\tau, \quad n \geq 2 \quad (5)$$

have been neglected.

II. Mathematical Formulation

The MHD flow of an electrically conducting visco-elastic fluid characterized by of Walters liquid (Model B′) confined between two vertical wavy walls in presence of a transverse magnetic field, radiation and temperature dependent heat source is considered. The x-axis is taken vertically upwards along the center of the channel and y-axis perpendicular to it. The wavy walls are represented by $y = \pm L + \varepsilon\cos(\lambda x)$ where, $\varepsilon \ll 1$ one of which is assumed to be isothermal and the other to be adiabatic.

The governing equations of the flow field are:

$$\frac{\partial \bar u}{\partial \bar x} + \frac{\partial \bar v}{\partial \bar y} = 0 \quad (6)$$

$$\bar u \frac{\partial \bar u}{\partial \bar x} + \bar v \frac{\partial \bar u}{\partial \bar y} = -\frac{1}{\rho}\frac{\partial \bar P}{\partial \bar x} + \nu\left(\frac{\partial^2 \bar u}{\partial \bar x^2} + \frac{\partial^2 \bar u}{\partial \bar y^2}\right) - \frac{K_0}{\rho}\left(\bar u \frac{\partial^3 \bar u}{\partial \bar x^3} + \bar u \frac{\partial^3 \bar u}{\partial \bar x \partial \bar y^2} + \bar v \frac{\partial^3 \bar u}{\partial \bar y \partial \bar x^2} + \bar v \frac{\partial^3 \bar u}{\partial \bar y^3} - 3\frac{\partial \bar u}{\partial \bar x}\frac{\partial^2 \bar u}{\partial \bar x^2} - \frac{\partial \bar v}{\partial \bar y}\frac{\partial^2 \bar u}{\partial \bar y^2} - \frac{\partial \bar u}{\partial \bar y}\frac{\partial^2 \bar v}{\partial \bar x^2} - \right.$$

∂u∂y∂2u∂x∂y−2∂v∂x∂2u∂x∂y+gβT−Te−σμe2H02ρu

(7)

$$\bar u \frac{\partial \bar v}{\partial \bar x} + \bar v \frac{\partial \bar v}{\partial \bar y} = -\frac{1}{\rho}\frac{\partial \bar P}{\partial \bar y} + \nu\left(\frac{\partial^2 \bar v}{\partial \bar x^2} + \frac{\partial^2 \bar v}{\partial \bar y^2}\right) - \frac{K_0}{\rho}\left(\bar u \frac{\partial^3 \bar v}{\partial \bar x^3} + \bar u \frac{\partial^3 \bar v}{\partial \bar x \partial \bar y^2} + \bar v \frac{\partial^3 \bar v}{\partial \bar y \partial \bar x^2} + \bar v \frac{\partial^3 \bar v}{\partial \bar y^3} - \frac{\partial \bar u}{\partial \bar x}\frac{\partial^2 \bar v}{\partial \bar x^2} - \frac{\partial \bar v}{\partial \bar x}\frac{\partial^2 \bar u}{\partial \bar y^2} - \frac{\partial \bar v}{\partial \bar x}\frac{\partial^2 \bar v}{\partial \bar x \partial \bar y} - \right.$$

3∂v∂y∂2v∂y2−2∂u∂y∂2v∂x∂y

(8)

$$\rho C_p\left(\bar u \frac{\partial \bar T}{\partial \bar x} + \bar v \frac{\partial \bar T}{\partial \bar y}\right) = k\left(\frac{\partial^2 \bar T}{\partial \bar x^2} + \frac{\partial^2 \bar T}{\partial \bar y^2}\right) - \frac{\partial q_{\bar x}{}^r}{\partial \bar x} - \frac{\partial q_{\bar y}{}^r}{\partial \bar y} + Q\left(\bar T_e - \bar T\right) \quad (9)$$

where $\nu = \frac{\eta_0}{\rho}$.

The boundary conditions are

$$\bar u = 0, \bar v = 0, \bar T = \bar T_0 \ \text{ at } \ y = -L + \bar\varepsilon\cos\bar\lambda\bar x$$

$$\bar u = 0, \bar v = 0, \ \frac{\partial \bar T}{\partial \bar y} = 0 \ \text{ at } \ y = L + \bar\varepsilon\cos\bar\lambda\bar x \quad (10)$$

We assume the Rosseland approximation (Brewstar [14]) for radiative heat flux, which leads to

$$q_{\bar x}{}^r = -\frac{4\sigma_1}{3\bar k}\frac{\partial \bar T^4}{\partial \bar x}, \quad q_{\bar y}{}^r = -\frac{4\sigma_1}{3\bar k}\frac{\partial \bar T^4}{\partial \bar y} \quad (11)$$

where σ_1 is the Stefan-Boltzmann constant and $\bar k$ is the mean absorption coefficient.

Taylor series expansion of $\overline{T}^4$ about $\overline{T}_e$, after neglecting higher order terms , is given by

$$\overline{T}^4 = 4\overline{T}_e{}^3\overline{T} - 3\overline{T}_e{}^4 \qquad (12)$$

We now introduce the following non-dimensional quantities:

$G_r = \frac{g\beta L^3(\overline{T}_0-\overline{T}_e)}{\nu^2}$ is the Grashoff Number,

$M^2 = \frac{\sigma\mu^2 H_0{}^2 L^2}{\rho\nu}$ is the Hartmann Number,

$P_r = \frac{\eta_0 C_p}{\kappa}$ is the Prandtl Number,

$x = \frac{\overline{x}}{L}, y = \frac{\overline{y}}{L}, u = \frac{L\overline{u}}{\nu}, v = \frac{L\overline{v}}{\nu}, P = \frac{\overline{P}L^2}{\rho\nu^2}, \theta = \frac{\overline{T}-\overline{T}_e}{\overline{T}_0-\overline{T}_e}, \alpha = \frac{QL^2}{\kappa}$ is the heat source parameter,

$\lambda = \overline{\lambda}L, \varepsilon = \frac{\overline{\varepsilon}}{L}$ is the non-dimensional amplitude ratio.

The non-dimensional form of equations (6) to (9) are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - K_1\left(u\frac{\partial^3 u}{\partial x^3} + u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + v\frac{\partial^3 u}{\partial x^2\partial y} - 3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} - 2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}\right) - Gr\theta - M^2 u \qquad (13)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - K_1\left(u\frac{\partial^3 v}{\partial x^3} + u\frac{\partial^3 v}{\partial x\partial y^2} + v\frac{\partial^3 v}{\partial x^2\partial y} + v\frac{\partial^3 v}{\partial y^3} - 2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial x^2} - 3\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) \qquad (14)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (15)$$

$$P_r\left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) = \left(1 + \frac{4}{3N}\right)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - \alpha\theta \qquad (16)$$

subject to the boundary conditions

$u = 0, v = 0, \theta=1$ on $y = -1+\varepsilon\cos(\lambda x)$

$u = 0, v = 0, \frac{\partial\theta}{\partial y} = 0,$ on $y = 1 +\varepsilon\cos(\lambda x)$ (17)

where $N = \frac{3\kappa\bar{k}}{4\sigma_1\bar{T}_e{}^3}$ is the radiation parameter and $K_1 = \frac{K_0}{\rho L^2}$ is the visco-elastic parameter.

Let $\omega = 1 + \frac{4}{3N}$ so that $N \to \infty \Longrightarrow \omega \to 1$ and the set of equations (13) to (17) represent flow and heat transfer in absence of radiation.

III. Method Of Solution

Assuming that the solution consists of a mean part and a perturbed part, we apply the perturbation scheme

$$u(x,y) = u_0(y) + \varepsilon u_1(x,y)$$
$$v(x,y) = \varepsilon v_1(x,y)$$
$$\theta(x,y) = \theta_0(y) + \varepsilon\theta_1(x,y)$$
$$P(x,y) = P_0(x) + \varepsilon P_1(x,y) \qquad (18)$$

to equations (13) to (16), where the perturbed quantities u_1, v_1, θ_1, P_1 are small compared with the mean quantities.

Comparing the coefficients of various powers of ε and neglecting those of second and higher powers of ε we get

$$\frac{d^2u_0}{dy^2} - M^2u_0 = -G_r\theta_0 - A \qquad (19)$$

where $A = -\frac{\partial P_0}{\partial x}$, a constant.

$$\omega\frac{d^2\theta_0}{dy^2} - \alpha\theta_0 = 0 \qquad (20)$$

to the zeroth order, and

$$u_0\frac{\partial u_1}{\partial x} + v_1\frac{du_0}{dy} = -\frac{\partial P_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - K_1\left(u_0\frac{\partial^3 u_1}{\partial x^3} + u_0\frac{\partial^3 u_1}{\partial x\partial y^2} + v_1\frac{d^3u_0}{dy^3} - \frac{\partial v_1}{\partial y}\frac{d^2u_0}{dy^2} - \frac{du_0}{dy}\frac{\partial^2 v_1}{\partial x^2} - \frac{du_0}{dy}\frac{\partial^2 u_1}{\partial x\partial y}\right) + G_r\theta_1 - M^2u_1 \qquad (21)$$

$$u_0\frac{\partial v_1}{\partial x} = -\frac{\partial P_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} - K_1\left(u_0\frac{\partial^3 v_1}{\partial x^3} + u_0\frac{\partial^3 v_1}{\partial x\partial y^2} - 2\frac{du_0}{dy}\frac{\partial^2 v_1}{\partial x\partial y} - \frac{\partial v_1}{\partial x}\frac{d^2u_0}{dy^2}\right) \qquad (22)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad (23)$$

$$P_r\left(u_0\frac{\partial\theta_1}{\partial x} + v_1\frac{d\theta_0}{dy}\right) = \omega\left(\frac{\partial^2\theta_1}{\partial x^2} + \frac{\partial^2\theta_1}{\partial y^2}\right) - \alpha\theta_1 \qquad (24)$$

to the first order.

The corresponding boundary conditions are

$$u_0 = 0, \theta_0 = 1 \quad at\ y = -1$$
$$u_0 = 0, \frac{\partial\theta_0}{\partial y} = 0 \quad at\ y = +1 \qquad (25)$$

$u_1 = -Real\left(u_0{}' e^{i\lambda x}\right), v_1 = 0, \theta_1 = -Real\left(\theta_0{}' e^{i\lambda x}\right)\ at\ y = -1$

$u_1 = -Real\left(u_0{}' e^{i\lambda x}\right), v_1 = 0, \frac{\partial \theta_1}{\partial y} = -Real\left(\theta_0{}'' e^{i\lambda x}\right)\ at\ y = +1$ (26)

where dashes denote differentiation with respect to y.

The actual boundary conditions (17) were transformed to the mean position $(y = \pm 1)$ of the walls to arrive at the boundary conditions (25) and (26) as is justified when $\varepsilon \ll 1$ i.e. when the amplitude ε of the disturbances is small compared with the disturbance wavelength (Lekoudis et al.[15], Tuck et al.[19]).

3.1. Solution

Solving equations (19) and (20) we obtain

$u_0 = D_3 e^{My} + D_4 e^{-My} + d_1 e^{Cy} + d_2 e^{-Cy} + d_3$ (27)

$\theta_0 = D_1 e^{Cy} + D_2 e^{-Cy}$ (28)

To solve equations (21) to (24), we assume that

$u_1 = -\frac{\partial \psi}{\partial y}, \quad v_1 = \frac{\partial \psi}{\partial x}$ (29)

On eliminating P_1 from equations (21) and (22) and on keeping in view the equation of continuity (23), equations (21), (22) and (24) yield

$u_0\psi_{xxx} + u_0\psi_{xyy} - u_0{}''\psi_x = \psi_{xxxx} + \psi_{yyyy} + 2\psi_{xxyy} - K_1\left(u_0\psi_{xxxxx} + 2u_0\psi_{xxxyy} + u_0\psi_{xyyyy} - u_0{}^{iv}\psi_x\right) - G_r\theta_{1,y} - M^2\psi_{yy}$ (30)

$P_r\left(u_0\theta_{1,x} + \psi_x\theta_0{}'\right) = \omega\left(\theta_{1,xx} + \theta_{1,yy}\right) - \alpha\theta_1$ (31)

Keeping in view (26), we assume general solutions for $\psi\ and\ \theta_1$ as follows:

$\psi(x, y) = Real[\sum_r(\psi_r\lambda^r)]e^{i\lambda x}$ (32)

$\theta_1(x, y) = Real\left[\sum_r(t_r\lambda^r)\, e^{i\lambda x}\right]$ (33)

Using (32) and (33) in equations (30) and (31) and equating coefficients of various powers of λ and neglecting those of second and higher powers of λ, we get the following sets of equations:

3.2. Zeroth order equations

$\psi_0{}^{iv} - M^2\psi_0{}'' = G_r t_0{}'$ (34)

$\omega t_0{}'' - \alpha t_0 = 0$ (35)

3.3. First order equations

$\psi_1{}^{iv} - M^2\psi_1{}'' = iK_1\left(u_0\psi_0{}^{iv} - u_0{}^{iv}\psi_0\right) + i(u_0\psi_0{}'' - u_0{}''\psi_0) + G_r t_1{}'$ (36)

$\omega t_1{}'' - \alpha t_1 = iP_r(u_0 t_0 + \theta_0{}'\psi_0)$ (37)

The corresponding boundary conditions are

$\psi_0(\pm 1) = 0, \psi_0{}'(\pm 1) = u_0{}'(\pm 1)$

$t_0(-1) = -\theta_0{}'(-1), t_0{}'(+1) = -\theta_0{}''(+1)$ (38)

$\psi_1{}'(\pm 1) = 0, \ \psi_1(\pm 1) = 0$

$t_1{}'(-1) = 0, \ t_1{}'(+1) = 0$ (39)

The solutions of equations (34) to 37) are

$\psi_0 = D_7 + D_8 y + D_9 e^{My} + D_{10} e^{-My} + d_4 e^{Cy} + d_5 e^{-Cy}$ (40)

$t_0 = D_5 e^{Cy} + D_6 e^{-Cy}$ (41)

$\psi_1 = D_{13} + D_{14}y + D_{15}e^{My} + D_{16}e^{-My} + d_{17}e^{(M+C)y} + d_{18}e^{(M-C)y} + d_{19}e^{(C-M)y} + d_{20}e^{-(C+M)y} + d_{25}e^{2Cy} + d_{26}e^{-2Cy} + d_{27}y^2 e^{My} + d_{29}y^2 e^{-My} + d_{30}ye^{My} + d_{31}ye^{-My} + d_{32}e^{Cy} + d_{33}e^{-Cy} + d_{34}ye^{Cy} + d_{35}ye^{-Cy} + d_{36}y^2 e^{Cy} + d_{37}y^2 e^{-Cy}$ (42)

$t_1 = D_{11}e^{Cy} + D_{12}e^{-Cy} + d_6 e^{(C+M)y} + d_7 e^{(M-C)y} + d_8 e^{(C-M)y} + d_9 e^{-(M+C)y} + d_{10}e^{2Cy} + d_{11}e^{-2Cy} + d_{12}ye^{Cy} + d_{13}ye^{-Cy} + d_{14}e^{Cy}\left(y^2 - \frac{y}{C}\right) + d_{15}e^{-Cy}\left(y^2 + \frac{y}{C}\right) + d_{16}$ (43)

From (29), (32), (33) and (40) to (43) and retaining up to the first power of λ in (29), we obtain the expressions for $u_1, v_1\ and\ \theta_1$ as follows:

$u_1 = -\psi_{0r}{}' cos\lambda x + \lambda\psi_{1i}{}' sin\lambda x$ (44)

$v_1 = -\lambda\psi_{0r} sin\lambda x - \lambda^2\psi_{1i} cos\lambda x$ (45)

$\theta_1 = t_{0r} cos\lambda x - \lambda t_{1i} sin\lambda x$ (46)

where ψ_{0r}=Real ψ_0, ψ_{1i}=Imag ψ_1, t_{0r}=Real t_0, t_{1i}=Imag t_1.

3.4. Skin friction

The non-dimensional skin friction coefficient σ_{-1}on the wavy walls $y = -1 + \varepsilon\, cos(\lambda x)$ is given by

$$\sigma_{-1} = \left(u'_0 + Re\varepsilon e^{i\lambda x}\left[u''_0 - \left(\psi''_0 + \lambda\psi''_1\right) - \lambda^2(\psi_0 + \psi_1) + K_1\{i\lambda u_0\left(\psi''_0 + \lambda\psi''_1\right) + i\lambda^3 u_0(\psi_0 + \lambda\psi_1) - i\lambda u''0\psi 0+\lambda\psi 1+2u'0\psi'0+\lambda\psi'1\right]_{y=-1} \quad (47)$$

The non-dimensional skin friction coefficient σ_1 on the wavy walls $y = 1 + \varepsilon\, cos(\lambda x)$ is given by

$$\sigma_1 = \left(u'_0 + Re\varepsilon e^{i\lambda x}\left[u''_0 - \left(\psi''_0 + \lambda\psi''_1\right) - \lambda^2(\psi_0 + \psi_1) + K_1\{i\lambda u_0\left(\psi''_0 + \lambda\psi''_1\right) + i\lambda^3 u_0(\psi_0 + \lambda\psi 1 - i\lambda u''0\psi 0+\lambda\psi 1+2u'0\psi'0+\lambda\psi'1\right]_{y=1} \quad (48)$$

3.5. Pressure drop

Using equations (18), (21), (29), (32) and (33) we obtain the fluid pressure at any point (x,y) as

$$\hat{P}(x,y) = -K'x + Re\,[\,\{\,\varepsilon\, i\, e^{i\lambda x})\}/\lambda]\, Z(y) + L \quad (49)$$

where L and K' are arbitrary constants and

$$Z(y) = (\psi_0''' + \lambda\psi_1''') - \lambda^2\left(\psi_0' + \lambda\psi_1'\right) - i\lambda\{-u_0'(\psi_0 + \lambda\psi_1) + u_0\left(\psi_0' + \lambda\psi_1'\right)\} - G_r(t_0 + \lambda t_1) - M^2(\psi_0 + \lambda\psi_1) + i\lambda K_1\{-u_0(\psi_0''' + \lambda\psi_1''') + \lambda^2 u_0\left(\psi_0' + \lambda\psi_1'\right) + u_0'''(\psi_0 + \lambda\psi_1) - u_0''(\psi_0' + \lambda\psi 1' + \lambda 2u0'\psi 0+\lambda\psi 1+u0'\psi 0''+\lambda\psi 1'' \quad (50)$$

The pressure drop $\hat{P}$ indicates the difference between the pressure at any point y in the flow field and that at the adiabatic wall y=1, with x- fixed and is given by

$$\hat{P} = \overline{P}(x,y) - \overline{P}(x,1)$$
$$= \frac{\varepsilon}{\lambda} Re\,[i\, e^{i\lambda x}\{Z(y) - Z(1)\}] \quad (51)$$

The constants are obtained but not given here due to brevity.

IV. Discussions

The purpose of this study is to bring out the effects of visco-elastic parameter on the free convective flow confined between two long vertical wavy walls as the effects of other flow parameters have been discussed by Kumar [31]. The visco-elastic effect is exhibited through the non-dimensional parameter K_1. The corresponding results for Newtonian fluid are obtained by setting K_1=0 and it is worth mentioning that these results show conformity with that of Kumar [31].

The expressions for u_1, v_1, and θ_1 are the first-order solutions or the disturbed parts due to waviness of the walls. The expression for the total velocity field (u, v) and the total temperature field θ may be obtained from (18) by using (27), (28) and from (44) to (46). The profiles of u_1 and v_1 are depicted against y in the figures 1, 2, 3 and 4, 5, 6 respectively to observe the visco-elastic effects. It is to be noted that the zeroth order quantities u_0, v_0 and θ_0 are not affected by the visco elastic parameter K_1. The numerical calculations are to be carried out for λx=π/4. This means that at the neighbourhood of λx=π/4.

It is evident from figure-1 that the first order vertical component of velocity profile u_1 increases with the increase of the visco-elastic parameter K_1 upto the central region of the channel and then decreases with no effect on the wall y=1 in comparison to that in Newtonian case. An opposite nature in the velocity profile u_1 is also observed from figure-1 for positive and negative values of the Grashof number G_r.

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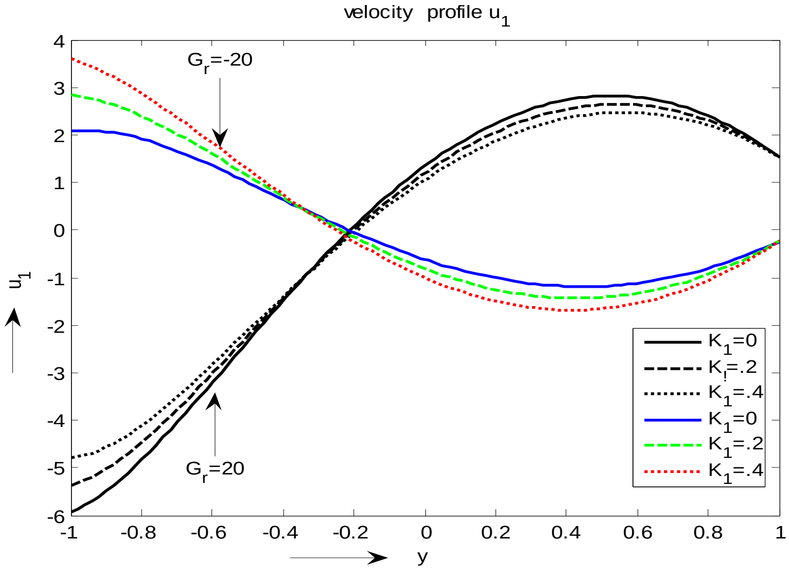

Fig-1: M=.5; λ=.01; λx=π/4; P_r=3; N=3; G_r=20; G_r= -20; α=10.

Figure-2 reveals that the velocity profile u_1 has a decreasing trend near the isothermal wall y=-1 and an increasing trend near the adiabatic wall y=1 with the increase in Grashof number G_r in both Newtonian and non-Newtonian cases.

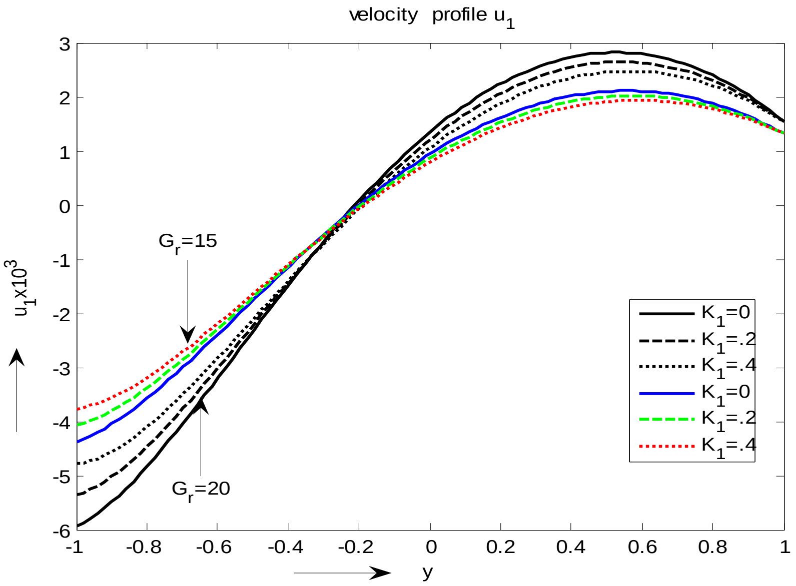

Fig-2: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; G_r=15; α =10.

Figure- 3 shows that the velocity profile u_1 increases with the increase of the visco-elastic parameter K_1 upto the central region of the channel and then decreases with no effect on the wall y=1irrespective of the value of the heat source parameter α. But the nature of the velocity profile $u_1$changes remarkably with the increase of the heat source parameter α in both Newtonian and non-Newtonian cases.

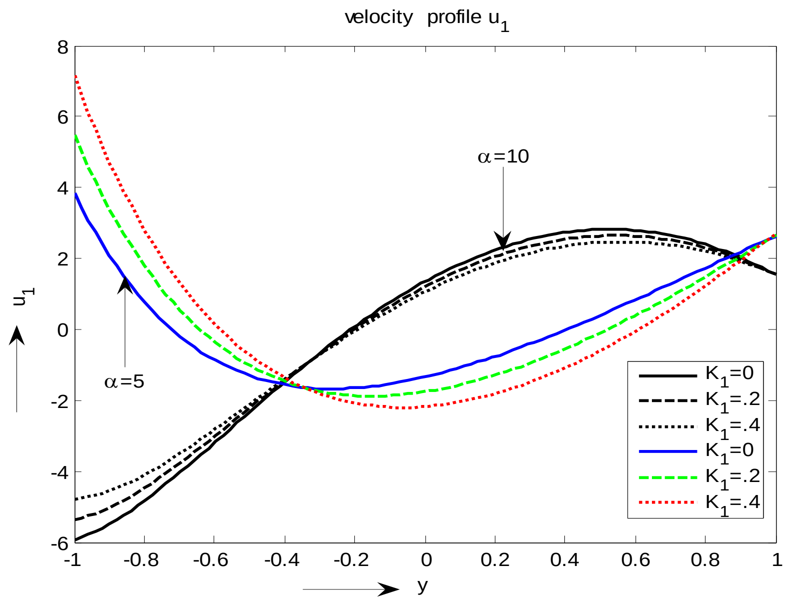

Fig-3: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; α =10; α =5;

Figures 4, 5 and 6 explain the nature of the horizontal component of the velocity profile v_1. It is observed from these figures that the velocity profile v_1 enhances with the visco-elastic parameter K_1 in comparison to that in Newtonian fluid flow. A U-turn is observed in the parabolic nature of the velocity profile v_1 in the cases of heated (G_r<0) and cooled (G_r>0) walls from figure-4. Figure-5 reveals a decreasing trend in the vertical component of velocity profile v_1 with the increase of Grashof number G_r in both Newtonian and non-Newtonian cases. An increasing trend in the vertical component of velocity profile v_1 with the increase of heat source parameter α is also observed from figure-6 in both Newtonian and non-Newtonian cases.

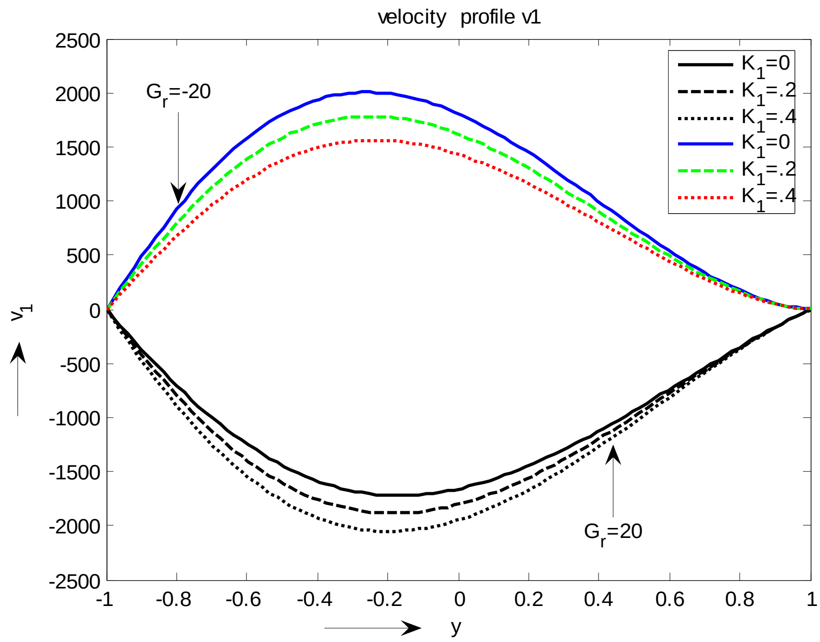

Fig-4: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; G_r=-20; α =10.

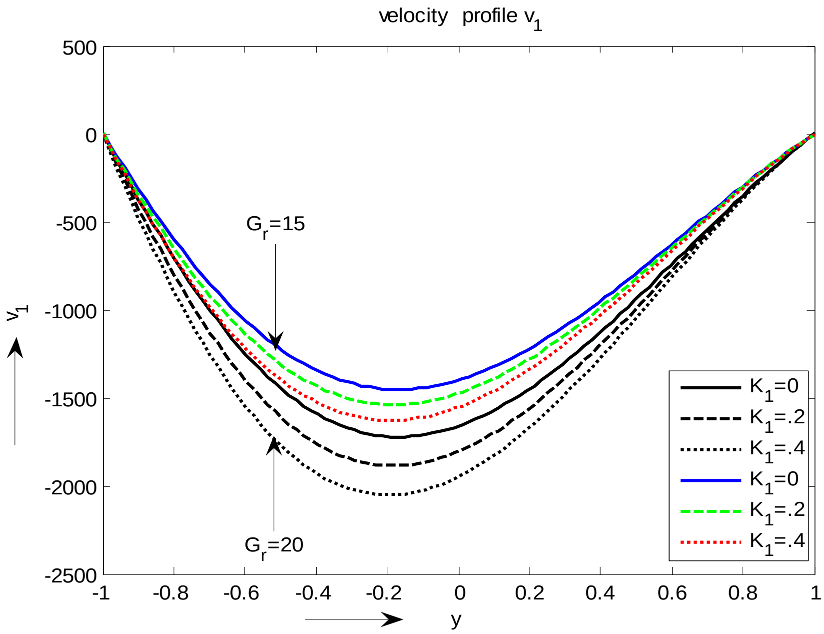

Fig-5: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; G_r=15; α =10.

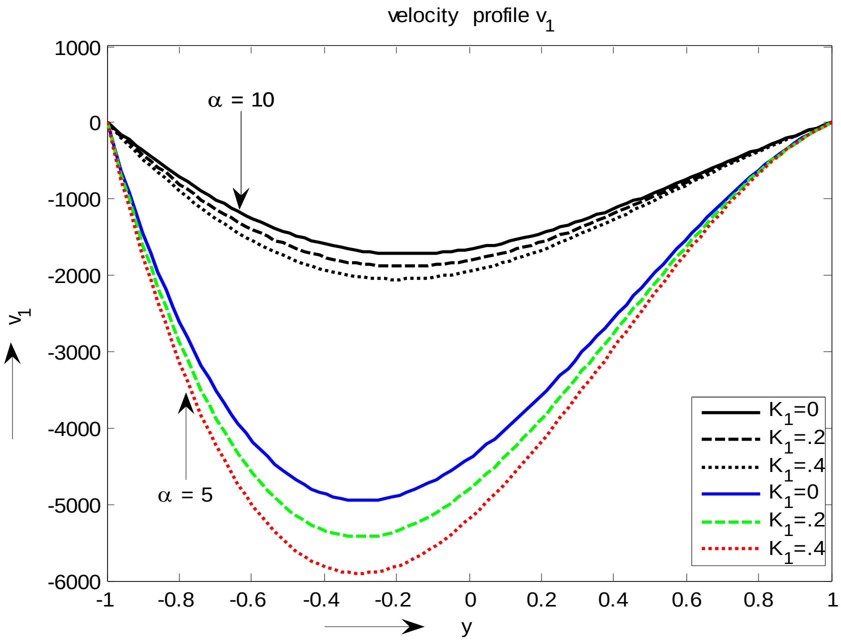

Fig-6: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; α =10; α =5.

Figures 7 and 8 explain the behavior of skin friction coefficient on the wavy walls y=-1 and y=1 respectively. It is evident from figure-7 that the skin friction coefficient on the isothermal wall y=-1 has an enhancing trend with the increasing of the visco-elastic parameter K_1 in comparison to that of Newtonian fluid flow for cooled wall (G_r>0) but an opposite trend is observed for heated wall (G_r<0).

It is clear from figure-8 that the skin friction coefficient on the adiabatic wall y=1 has an decreasing trend with the increase of the visco-elastic parameter K_1 in comparison to that of Newtonian fluid flow for cooled wall (G_r>0) but an opposite trend is observed for heated wall (G_r<0).

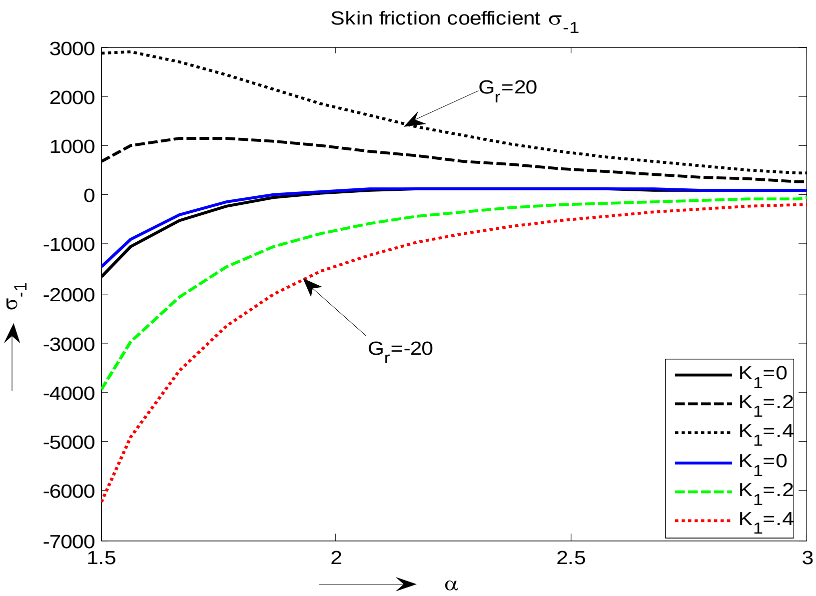

Fig-7: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; G_r=-20; y=-1; ε=.01.

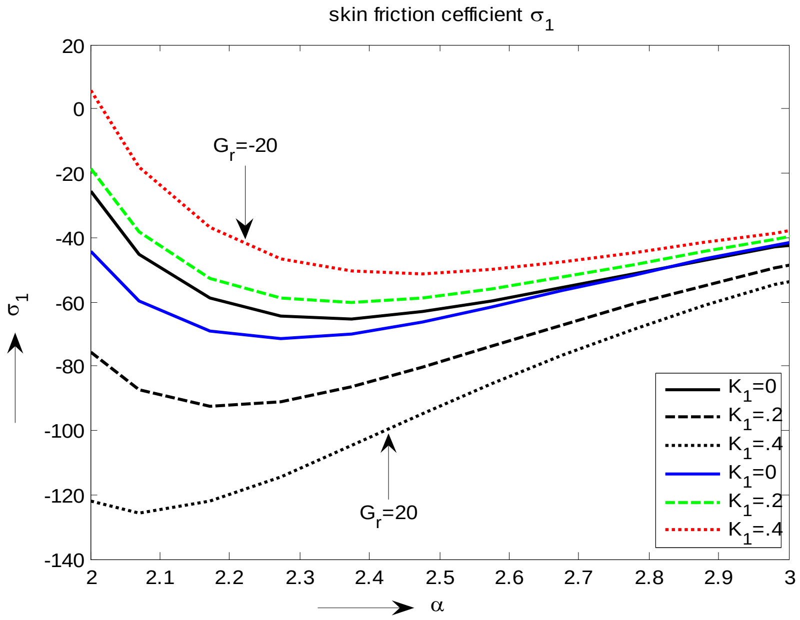

Fig-8: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; G_r=-20; y=1; ε=.01.

Figure-9 describes the effect of visco-elasticity on the pressure drop. It shows that the pressure drop increases with the increase of the visco-elastic parameter K_1 in both cooled (G_r>0) and heated (G_r<0) walls in the central region of the channel.

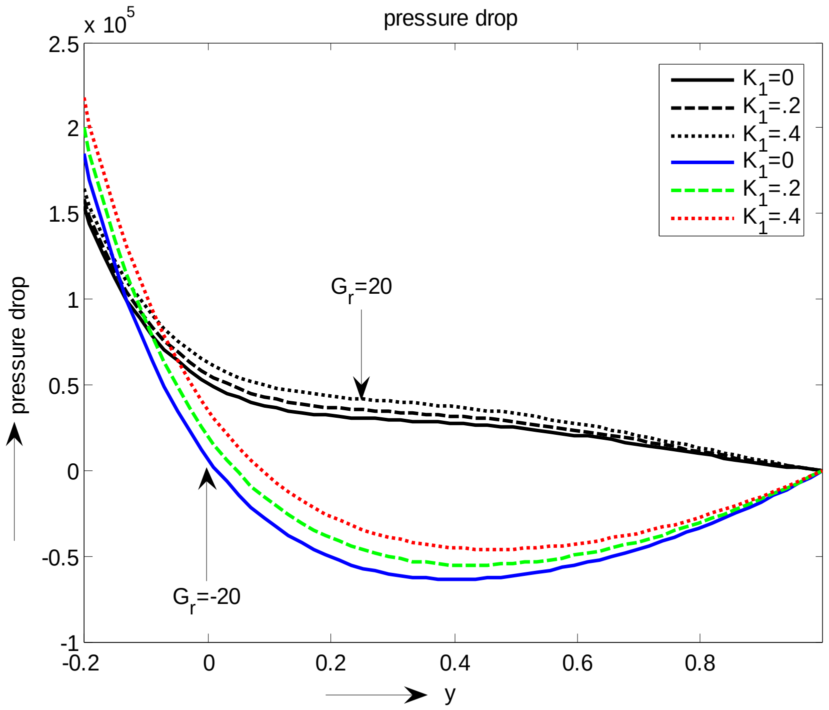

Fig-9: M=.5; λ =.01; λx= π /4; P_r =3; N=3; G_r=20; G_r=-20; ε=.01; α=10.

V. Conclusion

An analysis of the visco-elastic effects on free convective flow confined between two long vertical wavy walls has been presented for different values of visco-elastic parameter K_1 in combination of other flow parameters.

From this study, we make the following conclusions:

- The velocity field is considerably affected by the variation of visco-elastic parameter.
- A mixed type of effect of visco-elasticity is observed in the velocity component along the channel.
- The velocity component across the channel enhances with the rising of the visco-elasticity.

- The profile of pressure drop enhances by the rising of visco-elastic parameter in comparison with Newtonian fluid flow.
- The skin friction coefficient against the heat source/sink parameter increases with the increase of the visco-elastic parameter at the isothermal wall when it is cooled but an opposite trend is observed for heated wall in comparison to that in Newtonian case.
- The skin friction coefficient against the heat source/sink parameter decreases with the increase of the visco-elastic parameter at the adiabatic wall when it is cooled but an opposite trend is observed for heated wall in comparison to that in Newtonian case.
- The effect of visco-elastic parameter is not prominent in the temperature field.

References

[1]. Cess, R. D., "Interaction of thermal radiation with free convection heat transfer," Int. J. Heat Mass Transfer, vol. 9(1966), 1269-1277.
[2]. Arpaci,V. S., "Effects of thermal radiation on the laminar free convection from a heated vertical plate," Int. J. Heat Mass Transfer, Vol.11(1968), 871-881.
[3]. Cheng, E. H. and Ozisik, M. N., "Radiation with free convection in an absorbing emitting and scattering medium," Int. J. Heat Mass Transfer, vol.15 (1972), 1243-1252.
[4]. Hasegawa, S., Echigo, R. and Fakuda, K., "Analytic and experimental studies on simultaneous radiative and free convective heat transfer along a vertical plate. Proc. Jap. Soc.Mech. Eng., vol.38 (1972), 2873-2883.
[5]. Ozisik, M. N., "Radiation Transfer and Interactions with Conduction and Convection," Wiley-Interscience Publication USA, 1973.
[6]. Raptis, A., Massailas, C. and Tzivandis, G., "Hydro magnetic free convection flow through porous medium between two parallel plates," Phys. Lett. Vol. 90(A) (1982), 288-289.
[7]. Hossain, M. A. and Takhar, H. S., "Radiation effects on mixed convection along a vertical plate with uniform surface temperature," Heat and Mass Transfer, vol.31 (1996), 143-157.
[8]. Hossain, M. A. and Takhar, H. S., "Radiation-conduction interaction in mixed convection along rotating bodies," Heat and Mass Transfer, vol.33(1997), 201-208.
[9]. Hossain, M. A., Rees, D. A. S. and Pop, I., "Free convection-radiation interaction from an isothermal plate inclined at a small angle to the horizontal," Acta Mechanica, vol.127 (1998), 63-73.
[10]. Tak, S. S. and Kumar, H., "Heat transfer with radiation in MHD free convection flow confined between a vertical wavy wall and a flat wall," Bulletin of Pure and Applied Mathematics, vol.1 (2007), 2, 126-140.
[11]. Mohamed, R. A., Mahdy, A. and Hady, F. M., "Combined radiation and free convection from a vertical wavy surface embedded in porous media," Int. J. of Appl. Math and Mech., vol.4 (2008), 49-58.
[12]. Raptis, A. and Perdikis, C., "Radiation and free convection flow past a moving plate," Applied Mechanics and Engineering, vol.4 (1999), 817-821.
[13]. Ganeshan, P., Loganathan, P. and Soundalgekar, V. M., "Radiation effects on Flow Past an Impulsively Started Infinite Vertical Plate," Int. J. of Applied Mechanics and Engineering, vol.6 (2001), 719-730.
[14]. Brewster, M.Q., "Thermal Radiative Transfer and Properties," John Wiley and sons Inc, 1992.
[15]. Lekoudis, S.G., Neyfeh, A. H. and Saric, W. S., Compressible boundary layers over wavy walls. Phys. Fluids, vol.19 (1976), 514-519.
[16]. Shankar, P. N. and Sinha, U. N., "The Reyleigh problem for a wavy wall," J. Fluid Mech., vol.77 (1976), 243-256.
[17]. Bordner, G.L.," Nonlinear analysis of laminar boundary layer flow over a periodic wavy surface," Phys. Fluids, vol.21 (1978), 1471-1474.
[18]. Lessen, M. and Gangwani, S. T. "Effect of small amplitude wall waviness upon the stability of the laminar boundary layer," Phy. Fluids, vol.19 (1976), 510-513.
[19]. Tuck, E. O. and Kouzoubov, A., "A laminar roughness boundary condition," J. Fluid Mach., vol.300 (1995), 59-70.
[20]. Patidar, R. P. and Purohit, G. N., "Free convection flow of a viscous incompressible fluid in a porous medium between two long vertical wavy walls," Indian J. Math., vol. 40(1998), 67-86.
[21]. Vajravelu, K. and Sastri, K. S., "Free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, J. Fluid Mech., vol. 86(1978), 365-383.
[22]. Das, U. N. and Ahmed, N., " Free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and parallel flat wall," Indian J. Pure Appl. Math., vol. 23(1992), 295-304.
[23]. Sharma, P. K., "Fluctuating thermal and mass diffusion on unsteady free convective flow past a vertical plate in slip- surface with heat source/sink and viscous dissipation," Heat and Mass Transfer. Vol. 38(2005), 3, 213-220.
[24]. Tak, S. S. and Kumar, H., "MHD free convection flow with viscous dissipation in a vertical wavy channel, Ultra Scientist of Physical Sci., vol.18 (2006), 2M, 221-232.
[25]. Muthucumaraswamy, R. and Chandrakala, P., "Radiation heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction," J. of Appl.Mech. And Engng. Vol. 11(2006), 3, 639-646.
[26]. Nigam, S.D. and Singh, S.N., "Heat transfer by Laminar flow between parallel plates under the action of transverse magnetic field," Q. J. Mech. Appl. Math., vol.13 (1996), 85-87.
[27]. Mahdy, A., Mohamed, R. A. and Hady, F. M., "Natural convection from a vertical wavy plate embedded in porous media for power law fluids in presence of magnetic field," Int. J. of Appl. Math and Mech., vol. 5(2009), 8-15.
[28]. Mahdy, A., "Mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media (PST/PSC)," Int. J. of Appl. Math and Mech., vol.5 (2009), 88-97.
[29]. El-Aziz, M. A., "Radiation effect on the flow and heat transfer over an unsteady stretching sheet," International Communications in Heat and Mass Transfer. Vol. 36 (2009), 5, 521- 524.
[30]. Suneetha, S., Reddy, N. B. and Prasad, V. R., "Radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink," JAFM, vol.4(2010), 1, 107-113.
[31]. Kumar, H., "Heat transfer with radiation and temperature dependent heat Source in MHD free convection flow confined between two vertical wavy walls," Int. J. of Appl. Math and Mech. Vol.7(2011), 2, 77-103.
[32]. Basu, B., Prasad, V. R. and Reddy, N. B., "Radiation and mass transfer effects on transient free convection flow of dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux," JAFM, vol.4(2011), 1, 15-26.

[33]. Reddy, M. G. and Reddy, N. B., "Mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium," JAFM, vol.4(2011), 2, 7-11.
[34]. Sandeep, N., Reddy, A. V. B. and Sugunamma, V., "Effect of Radiation and Chemical Reaction on Transient MHD Free Convective Flow over a Vertical Plate Through Porous Media," Chem. and Process Engng Res. Vol. 2(2012), 1-9.
[35]. Bala, P., Reddy, A. N. B. and Suneetha, S., "Radiation effects on MHD flow past an exponentiallly accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source," JAFM, vol.5(2012), 3, 119-126.
[36]. Choudhury, R. and Das, S. K., "Visco-elastic effects on free convective flow confined between a long vertical wavy wall and a parallel flat wall of equal transpiration," Int. J. of Appl. Engg. Research, Vol. 7(2012), 7, 691-706.
[37]. Choudhury, R. and Das, S. K., "Visco-elastic free convective flow and heat transfer past an oscillating porous plate in the slip flow regime," Adv. in Theo. and Appl. Math., Vol. 7(2012), 4, 403-416.
[38]. Choudhury, R. and Das, S. K., "Visco-elastic MHD fluid flow over a vertical plate with Dufour and Soret effect," Int. J. of Sci. and Engg. Res., Vol. 4(2012), 7, 11-17.
[39]. Rao, J. A., Sivaiah,S. and Raju, R. S., "Chemical effects on an unsteady MHD free convection fluid past a semi-infinite vertical plate embedded in a porous medium with heat absorption," JAFM, vol.5(2012), 3, 63-70.
[40]. Choudhury, R. and Das, S. K., "Visco-elastic unsteady MHD flow between two horizontal parallel plates with hall current,"J. of Math., Vol. 5(2013), 1, 20-28, doi 10.9790/5728-0512028.
[41]. Choudhury, R. and Das, S. K., "Mixed convective visco-elastic MHD flow with ohmic heating," Int. J. of Comp. Appl., Vol.68 (2013), 10, 7-13. Impact Factor:0.814(2011), doi10.5120/11613-7004
[42]. Choudhury, R. and Das, S. K., "Visco-elastic mhd oscillatory horizontal channel flow and heat transfer with heat source," Appl. Math. and Comp. Sci., Vol. 4(2013), 3, 149-206.
[43]. Choudhury, R. and Das, S. K., "Visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer," J. Appl. Fluid Mech. (JAFM), Vol.7 (2014), 4, 603-609.
[44]. Das, S. K., "Radiation and soret effect on visco-elastic mhd oscillatory horizontal channel flow with heat and mass transfer in presence of chemical reaction and heat source," Int. J. of Engg. Res. and Appl., Vol. 6(2016), 8, (Part -4), 39-54.
[45]. Choudhury, R. and Das, S. K.," Visco-elastic mhd flow with heat and mass transfer past a porous plate in presence of chemical reaction and heat generation," Int. Res. J. of Math., Engg and IT (IRJMEIT),Vol.3(2016), 8, 28-43.
[46]. Walters, K., "The motion of an elastico-viscous liquid contained between co-axial cylinders (II)," Quart. J. Mech. Appl. Math. Vol.13 (1960), 444-461.
[47]. Walters, K., "The solution of flow problems in the case of materials with memories," J. Mecanique. Vol.1 (1962), 473-478.

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